Slowly, rotating non-stationary, fluid solutions of Einstein's equations and their match to Kerr empty space-time

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Abstract

A general class of solutions of Einstein's equation for a slowly rotating fluid source, with supporting internal pressure, is matched using Lichnerowicz junction conditions, to the Kerr metric up to and including first order terms in angular speed parameter. It is shown that the match applies to any previously known non-rotating fluid source made to rotate slowly for which a zero pressure boundary surface exists. The method is applied to the dust source of Robertson-Walker and in outline to an interior solution due to McVittie describing gravitational collapse. The applicability of the method to additional examples is transparent. The differential angular velocity of the rotating systems is determined and the induced rotation of local inertial frame is exhibited.

1 Introduction

In the period since the discovery of the Kerr [1] metric which describes analytically, the asymptotically flat, vacuum gravitational field outside a rotating source in terms of Einstein's field equations, there have been many attempts to find closed interior solutions which match the exterior smoothly. In general terms attempts to find solutions have proved unsuccessful as has been described by Pichon and Lynden-Bell [2]. One difficulty has been the considerable mathematical complexity in solving Einstein's equations, see for example, Krasinski [3], Chinea & Gonzalez-Romero [4]. This has led to an 'embarrassing hiatus' according to Bradley *et al* [5] in the number of potential interior solutions available for matching which in turn has contributed to a lack in the development in the theory of differentially rotating fluid bodies in general relativity. Even for the case of the remarkable and much quoted, Wahlquist [6] closed form interior there is no possible fit to the Kerr exterior as has been recently been shown by Bradley *et al* [5]. Only for the important case of thin super-massive rotating discs, supported by internal pressure have analytic sources for the Kerr metric been found (Pichon and Lynden-Bell [2]). Yet it is important to develop further the relativistic theory of rotation since it has considerable potential application in astrophysics, for example, in the description of the gravitational collapse of rotating matter, quasars, or potential sources for gravitational radiation.

To this end one way forward is perhaps to adopt a linearised perturbation technique wherein, in the first instance, the interior source is rotating only very slowly. Following the successful match to the Kerr exterior one would proceed to develop higher order perturbation methods to describe sources with higher angular velocity. The use of perturbation techniques in general relativity are of course common. In the context of rotation they have been applied successfully by Hartle [7] to

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